

# The *Ultimate* Formula Sheet for ACT Math

The ACT does not provide any formulas. Be prepared by making sure to have these ones memorized.

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## Fractions, Decimals, & Percentages: (for this section, r is the percent in decimal form)

$$\text{Fraction} = \frac{\text{part}}{\text{whole}}; \text{Percent} = \frac{\text{part}}{100}$$

$$\text{Percent Increase or Decrease: } \frac{|\text{old} - \text{new}|}{\text{old}} \times 100\%$$

Increase by a percent: multiply by  $(1+r)$

Decrease by a percent: multiply by  $(1-r)$

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$$\text{Simple Interest: } A = P(1+rt)$$

$$\text{Interest Compounded Annually: } A = P(1+r)^t$$

Interest Compounded n times per year:

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

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## Rates, Ratios, & Proportions:

General form of a conversion factor:

$$\left( \frac{\text{ending\_units}}{\text{starting\_units}} \right)$$

$$\text{Example: } 10\text{feet} \left( \frac{12\text{inches}}{1\text{foot}} \right) = 120\text{inches}$$

$$\begin{aligned} & (\text{Concentration of A} \times \text{Volume of A}) \\ & + (\text{Concentration of B} \times \text{Volume of B}) \\ & = \text{Final concentration (Vol. of A} + \text{Vol. of B)} \end{aligned}$$

$$\text{Distance} = \text{Rate} \times \text{Time}$$

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## Exponents, Roots, & Polynomials:

$$\text{Multiplication Rule for Exponents: } a^b \cdot a^c = a^{b+c}$$

$$\text{Division Rule for Exponents: } \frac{a^b}{a^c} = a^{b-c}$$

$$\text{Power Rule for Exponents: } (a^b)^c = a^{bc}$$

$$\text{Negative Exponents: } a^{-b} = \frac{1}{a^b}$$

$$\text{Fractional Exponents: } a^{\frac{b}{c}} = \sqrt[c]{a^b} \text{ or } (\sqrt[c]{a})^b$$

$$i = \sqrt{-1}; i^2 = -1; i^3 = -i; i^4 = 1$$

$$i^{4n} = 1; i^{4n+1} = i; i^{4n+2} = -1; i^{4n+3} = -i$$

$$\text{Complex Conjugates: } (a+bi)(a-bi)$$

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## Parabolas:

$$\text{Standard Form: } f(x) = ax^2 + bx + c;$$

$$\text{vertex} = \left( -\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right);$$

y-intercept = c;

$$\text{x-intercepts} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Sum of solutions} = \frac{-b}{a}$$

$$\text{Discriminant} = b^2 - 4ac; \text{Pos} = 2 \text{ real roots}$$

Zero = 1 real root; Neg = 2 imaginary roots

$$\text{Factored Form: } f(x) = a(x-m)(x-n);$$

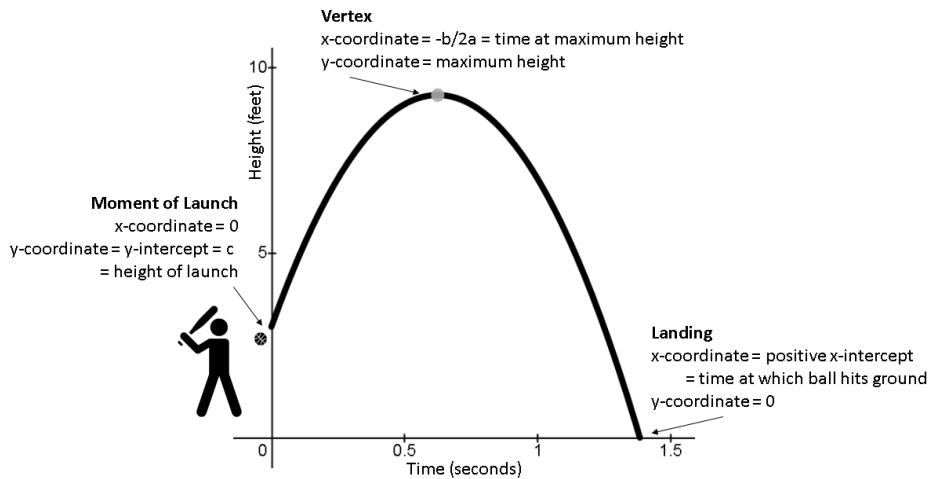
x-intercepts are m and n;

$$\text{x-coordinate of vertex} = \frac{m+n}{2}$$

$$\text{Vertex Form: } f(x) = a(x-h)^2 + k;$$

vertex = (h,k)

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Difference of Squares:  $a^2 - b^2 = (a+b)(a-b)$

Sum of Cubes:  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$       Difference of Cubes:  $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Perfect Square Trinomial:  $a^2 + 2ab + b^2 = (a+b)^2$  and  $a^2 - 2ab + b^2 = (a-b)^2$

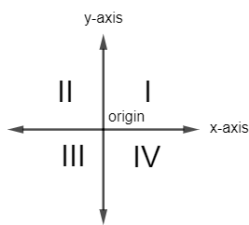
Completing the Square:  $x^2 + bx + \left(\frac{b}{2}\right)^2 = (x + \frac{b}{2})^2$

**Graphing Lines:**

Slope Formula:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope of horizontal line = 0

Slope of vertical line = undefined



Standard Form:  $Ax + By = C$

Slope-Intercept Form:  $y = mx + b$

Point-Slope Form:  $y - y_1 = m(x - x_1)$

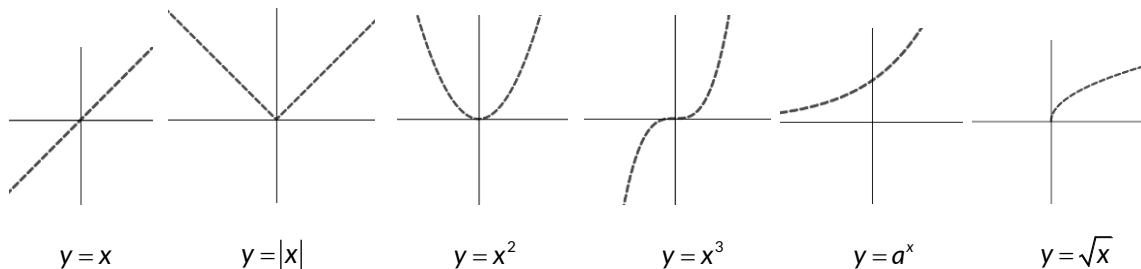
Distance Formula:  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula:  $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Parallel lines: equal slopes

⊥ Lines: slopes are opposite reciprocals

**Parent Graphs & Transformations:**



**Transformation**

$f(x)+k$

$f(x)-k$

$f(x+h)$

$f(x-h)$

$-f(x)$

$cf(x)$

$\frac{1}{c}f(x)$

**Visual effect**

Shift up by k units

Shift down by k units

Shift left by h units

Shift right by h units

Reflect over the x axis (flip upside down)

Stretch vertically by a factor of c (becomes skinnier)

Shrink vertically by a factor of c (becomes fatter)

**Data & Probability:**

$$\text{Average} = \frac{\text{sum\_of\_items}}{\text{number\_of\_items}}$$

Median = center data point

Mode = most frequent data point

Range = maximum - minimum

$$\text{Probability} = \frac{\text{desired\_outcomes}}{\text{possible\_outcomes}}$$

Probability that independent events A and B will both happen:  $P(A \cap B) = P(A) \times P(B)$ Probability that either A or B will happen:  
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 

$$\text{Expected Value: } E(x) = \sum_{i=1}^n x_i \cdot P(x_i)$$

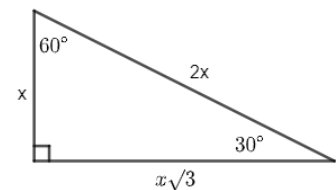
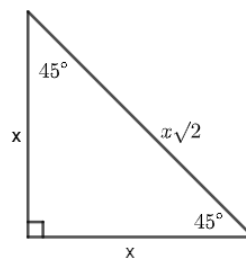
**Angles:**Vertical  $\angle$ 's are  $\cong$  $\angle$ 's that form a linear pair are supplementary (add up to  $180^\circ$ ) $\angle$ 's that form a circle add up to  $360^\circ$ When  $\parallel$  lines are cut by a transversal, all acute  $\angle$ 's are  $\cong$  and all obtuse  $\angle$ 's are  $\cong$ **Triangles:**

$$\text{Area of a Triangle: } A = \frac{1}{2}bh$$

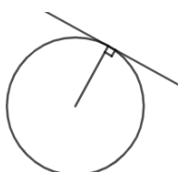
The three  $\angle$ 's of a  $\Delta$  add up to  $180^\circ$ An exterior  $\angle$  is equal to the sum of the two remote interior  $\angle$ 's

$$\text{Pythagorean Theorem: } a^2 + b^2 = c^2$$

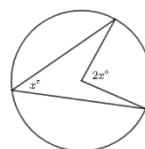
Pythagorean Triples: 3-4-5 and 5-12-13

**Special Right Triangles:****Circles:**

$$\text{Area of a Circle: } A = \pi r^2$$

A radius and tangent make a right  $\angle$ 

$$\text{Circumference of a Circle: } C = 2\pi r$$

A central  $\angle$  is double the inscribed  $\angle$

$$\frac{x}{360} = \frac{\text{arc}}{\text{circumference}} \quad \text{and} \quad \frac{x}{360} = \frac{\text{sector}}{\text{area\_of\_circle}} \quad \text{where } x = \text{central angle}$$

**Polygons: (for this section, n is the number of sides)**

Area of a Rectangle:  $A = lw$

One int.  $\angle$  of a regular polygon:  $\frac{180(n-2)}{n}$

Area of a trapezoid:  $\frac{1}{2}(b_1 + b_2)h$

# of diagonals:  $\frac{n(n-3)}{2}$  (convex only)

Sum of the exterior angles:  $360^\circ$

Sum of the interior angles:  $180(n-2)$

**Properties of Parallelograms:**

1. Opp sides are  $\parallel$  and  $\cong$

5. Diagonals bisect each other

2. Opp  $\angle$ 's are  $\cong$

→ If they are  $\cong$  it is a rectangle

3. Consec  $\angle$ 's are supplementary

→ If they are  $\perp$  it is a rhombus

4. Each diagonal forms a pair of  $\cong \Delta$ 's

6.  $\text{Area} = \text{base} \times \text{height}$

**Solids:**

Volume of a Rectangular Prism (Box):  $V = lwh$

Volume of a Sphere:  $V = \frac{4}{3}\pi r^3$

Surface Area of a Box:  $SA = 2(lw + lh + wh)$

Volume of a Cone:  $V = \frac{1}{3}\pi r^2 h$

Volume of a Cylinder:  $V = \pi r^2 h$

Volume of a Pyramid:  $V = \frac{1}{3}lwh$

Surface Area of a Cylinder:  $SA = 2\pi r^2 + 2\pi rh$

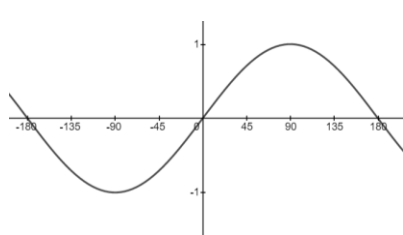
**Trigonometry:**

$$\sin = \frac{\text{opp}}{\text{hyp}} \quad \cos = \frac{\text{adj}}{\text{hyp}} \quad \tan = \frac{\text{opp}}{\text{adj}} \quad \csc(x) = \frac{1}{\sin(x)} \quad \sec(x) = \frac{1}{\cos(x)} \quad \cot(x) = \frac{1}{\tan(x)}$$

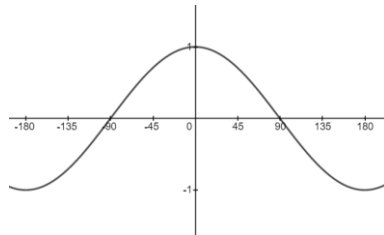
$$360^\circ = 2\pi \text{ radians} \quad \tan x = \frac{\sin x}{\cos x} \quad \sin^2 x + \cos^2 x = 1 \quad \sin(x) = \cos(90 - x)$$

Law of Sines:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

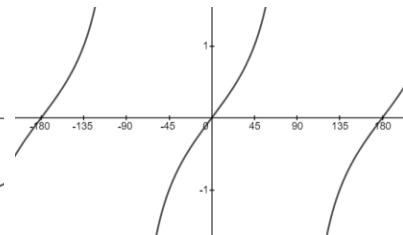
Law of Cosines:  $a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$



$y = \sin(x)$



$y = \cos(x)$



$y = \tan(x)$

If  $y = A\sin(Bx - C) + D$  (also for cos, csc, and sec)

Amplitude:  $|A|$       Period:  $\frac{2\pi}{B}$       Phase Shift:  $\frac{C}{B}$       Vertical Shift:  $D$

If  $y = A\tan(Bx - C) + D$  (also for cot)

Amplitude: none      Period:  $\frac{\pi}{B}$       Phase Shift:  $\frac{C}{B}$       Vertical Shift:  $D$

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**Sequences and Series:** where  $a_1$  = first term,  $n$  = number of terms,  $d$  = common difference,  $r$  = common ratio

Arithmetic sequence:  $a_n = a_1 + (n-1)d$

Geometric sequence:  $a_n = a_1 r^{n-1}$

Sum of an arithmetic series:  $S_n = \frac{n}{2}(a_1 + a_n)$

Sum of a geometric series:  $S_n = \frac{a_1(r^n - 1)}{r - 1}$

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**Logarithms:**

If  $\log_b a = x$ , then  $b^x = a$

$$\log_b a = \frac{\log a}{\log b}$$

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**Vector Addition:**  $\vec{a} + \vec{b} = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

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**Matrix Multiplication:** Only possible when columns of first = rows of second

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \times \begin{pmatrix} E & F \\ G & H \end{pmatrix} = \begin{pmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{pmatrix}$$

Determinant of  $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = AD - BC$

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**Conic Sections:**

Circle:  $(x-h)^2 + (y-k)^2 = r^2$ , where  $(h,k)$  is the center and  $r$  is the radius

Ellipse:  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  where  $(h,k)$  is the center,  $2a$  is the horizontal axis, and  $2b$  is the vertical axis

Horizontal Ellipse:  $a^2 = b^2 + c^2$  Vertical Ellipse:  $b^2 = a^2 + c^2$  where  $c$  is the distance from center to focus

Horizontal Hyperbola:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$  Vertical Hyperbola:  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

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