Mindset Strategy Practice

for

SAT Math

Chapter 6: Mathematical Models

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Chapter 6 – Mathematical Models
Pre-Test

Directions: Try these five problems without using any resources to help you and record the number of questions you answer correctly. The goal is to give you a snapshot of your current knowledge on this topic and a preview of the chapter to come. Use a calculator only if you need to.

 # Correct:

1.) The total income from taxes in a certain town is \$540 million. If \$235 million of that amount comes from property taxes and *y* comes from all other sources, which of the following represents the value of *y* in millions of dollars?

A) $y - 540 = 235$

B) $y - 235 = 540$

C) $y = 235 - 540$

D) $y = 540 - 235$

2.) An airplane initially cruising at an altitude of 10,000 feet begins to climb at a rate of 10 feet per second. Which of the following functions represents the airplane's altitude a, in feet, s seconds after the airplane begins its climb?

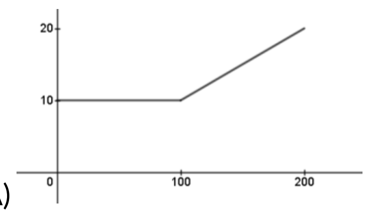
A) $a(s) = 10000 + 10s$

B) $a(s) = 10000s + 10$

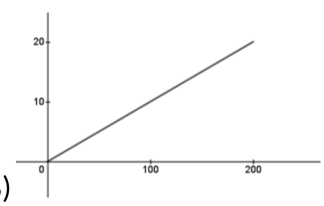
C) $a(s) = 10000s - 10$

D) $a(s) = 10000 - 10s$

3.) A text messaging plan charges \$10 per month for the first 100 texts and \$0.10 for each text after the first 100 sent in that month. Which of the following graphs represents the cost, *y*, of sending *x* texts in a month on the standard *xy*-plane?



A)



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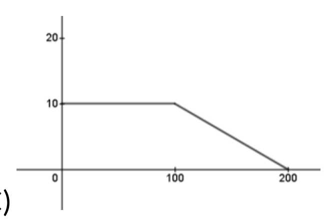
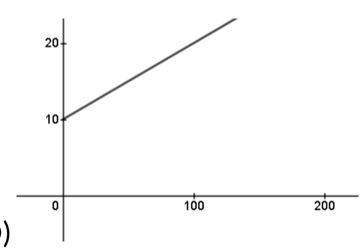


C)



D)

4.) Which of the following options best describes the pattern in the table below?

x	y
0	1031
1	1000
2	969
3	938

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

5.) The quadratic function $h(x) = -16x^2 + 24x + 5$ models the height above the ground, h, in feet, of a cannon ball x seconds after it was launched from a cannon. If $y = h(x)$ is graphed on the standard xy-plane, which of the following represents the real world meaning of the y-coordinate of the vertex.

A) The time at which the cannon ball reaches its maximum height

B) The maximum height of the cannon ball

C) The height above the ground at which the cannon ball was launched

D) The time at which the cannon ball reaches the ground

Dear Student,

By the time Angie came to me for SAT prep, I already knew her whole family. Angie was the youngest of three daughters, and I had already tutored her older sisters. Angie's oldest sister, Deborah, got a fantastic score on her SAT, attended a prestigious college, is currently in medical school, and is on her way to becoming a neurosurgeon. The next sister, Leah, was a very bright student and serious worker, but struggled with dyslexia. I remember at my last tutoring session with Leah, we stacked all her SAT prep books one on top of the other and created a tower about three feet high. Leah was so proud of how hard she worked that she took a photo of the SAT prep book tower and texted it to all her friends. Angie was different. It's not that she wasn't smart – she just wasn't the academic superstar that her oldest sister was. And it's not that she wasn't a hard worker – she just didn't have any enthusiasm for SAT prep. It wasn't even that her parents expected her to be like her sisters, but they did expect her to do her best, and that, to be frank, wasn't happening.

Angie's parents signed her up for classes, tutoring, and SAT prep at my tutoring center during the summer between her sophomore and junior year. They told her to consider SAT prep as her "summer job." I loved having Angie at the center that summer. She was always polite and happy to chat about what was going on in her life. She got along well with the other students. But as soon as we started to work, she withdrew. She did all the activities and assignments but was clearly not into it. She took four practice tests that summer and improved her score from a 1060 to an 1160, but I knew she was still scoring below her potential. When we went over a test together, she would get angry with herself for all the "dumb mistakes." Worst of all, when she took the official SAT in the fall of her Junior year, her score went right back down to a 1060.

Angie's problem, aside from comparing herself to her older sisters, was a lack of metacognition. The psychological term metacognition literally means *thinking about thinking*. It means having self-awareness about your level of understanding, ability to focus, and attention to detail. I know I'm working with a student with strong metacognitive skills when they take a distinct pause after we go over a question and before we move on to the next one. I can almost see the student mentally reviewing the steps one more time to make sure they have full understanding. Another time I know a student has great metacognitive skills is when they ask, "I know I got this question right, but can we go over it anyway?" Metacognition is what enables you to appreciate the difference between an earned point and a lucky guess. When you are taking the SAT, metacognition is what warns you that your focus is slipping and that you need to use a strategy, such as underlining important words or writing out more steps, to avoid careless errors.

When Angie came to SAT prep, she went through all the motions, but her negative mindset got in the way of her metacognition. When we went over the missed math problems on her practice tests, there were several that she knew exactly how to do but had still gotten incorrect. When she didn't know how to do a problem, I could easily show her, but the prevalence of careless errors was a trickier dilemma to address. I encouraged her to mark up the questions to make sure she was attending to the words that described concepts and relationships, to write down more steps and sometimes just her thoughts, and to always reread the question a third time before moving on to the next problem. All three of these strategies were meant to improve her metacognition. In other words, these strategies aimed to help her be more aware of her level of understanding when she read a question and the diligence of her thought process as she worked through the steps. Rereading the question before moving on to the next one required an acknowledgement that she could make careless errors and a concerted effort to find and correct those errors.

Angie is smart and a hard worker, but she has a lot of negative emotions, probably because she can't avoid comparing herself to her ultra-successful family members. Those negative emotions make her not want to think too hard during the test. The good news for Angie is that she's still a high school junior and has several opportunities to continue studying and retake the test. She is planning on doing just that and trying out the ACT as well. Angie's family absolutely loves her and accepts her for who she is. At the same time, her parents know her well enough to know that she hasn't achieved her best score yet. Angie needs to learn to accept herself, too, and to understand that she doesn't need to be the same as her sisters to be her own fantastic self. If she can get past her fear of not living up to the standards set by her sisters, she should be able to remove the emotional block that is interfering with her metacognition.

There are two lessons that I want you take from Angie's story. First, always remember you are pushing for your own personal best. If you compare yourself to a family member, friend, or that smart kid in your class, you're likely to be setting your goal too high or too low and stressing yourself out in the process. In order to set your own standard, take a practice test to set your baseline score. Next, give yourself time to improve your overall SAT score by about 200 points. Give yourself deadlines, such as the PSAT in October of your junior year, and study hard for them. Give yourself a break for a couple of weeks, so you don't get burned out. Then hit the books again several weeks before the next test. When you do hit your goal, you've earned the right to be proud of yourself, no matter how it compares to that annoying kid in the front row of your chemistry class.

The second thing I want you to remember is the importance of metacognition. You are the only one capable of monitoring your own thinking, and to achieve your goals, you'll have to do exactly that. Just going through the motions is never enough. If you notice that you tend to lose focus or make careless errors, try out a few different strategies to address those problems and pay attention to which ones work for you and which don't. Make a list of the strategies that are best for you and review it before every practice and official test.

Metacognition will be especially important for the types of questions in this chapter. Questions about mathematical models are frequently done without touching your calculator or writing down any steps. They are especially difficult to teach because you have to show someone how to think. When you get one of these questions wrong, don't just move on to the next one: analyze your error. Was there an important word that you disregarded? Did you read all the options? Were you unaware of an important pattern? Did you fall for a trap? If so, what was the nature of the trap? When you do pinpoint the source of your error, make a note of it to help you remember what your weakness is. If you are aware of a weakness, then you've taken the first step toward turning that weakness into a strength. The next steps are to maintain a positive attitude and keep practicing. So what are you waiting for? Chapter 6 awaits!

Sincerely,

Ms. Krey

Strategy

A Mathematical model is an equation or a graph that represents a real world situation. When you create a mathematical model, you take a real world situation with observable inputs and results then create an equation, graph, or table that will calculate predicted outputs from any inputs. The SAT just loves mathematical models. They will ask you to interpret them, apply them, and create them over and over again. Many of the word problems I've already discussed could fit under this category, including:

- Percent increase or decrease problems in chapter 2
- Interest word problems in chapter 2
- Ratio word problems in chapter 3
- Mixture word problems in chapter 3
- D=RT word problems in chapter 3
- Systems of equations word problems in chapter 5
- Inequality word problems in chapter 5

The problem with the real world is that it's messy. The mathematical models that statisticians, meteorologist, financial analysts, and other professionals create to predict what might happen in the future are incredibly complicated and frequently incorrect. The models we use to predict the path of a hurricane, for example, have taken years to develop and still don't always get it right. As a result, the questions you see on the SAT often feel forced and overly simplified. Instead of being real world questions, they're more like real world wannabes. Consider the following question:

Example: Larry and Harry go to a bistro for lunch together. The meal that Harry orders costs $1 more than the meal that Larry orders. There is no tax, and they both leave a 20% tip. If Larry's meal cost x dollars, which of the following functions represents the total dollars they paid, t, in terms of x?

A) $t(x) = 0.20(x + 1)$

B) $t(x) = (2x + 1) + .2x$

C) $t(x) = 1.2(2x + 1)$

D) $t(x) = 0.2x + 0.2(x + 1)$

The makers of the SAT want you to believe this is a real world question. But, seriously, who in the real world goes out to lunch with a friend and creates an equation to split the bill? Larry didn't have to think in terms of x when he paid the bill: he knew what the price was when he ordered his meal. This isn't a real world question, but rather it's a real world wannabe. Nevertheless, you still have to do your best to answer the question. Set aside your disbelief that Harry knew he ordered something exactly $1 more than his friend, without actually knowing what the price was, and focus on matching the description in the problem to the correct formula.

Explanation: In order to approach this problem systematically, I would show my thinking something like this:

Larry = x

Harry = x+1

Together = 2x+1

Together with tip= 1.2(2x+1)

The hardest part of this question was remembering that when you add a 20% tip, you need to multiply by 1.2. Think of it this way: if you simply multiply by 0.2, the number will get much smaller. But when you add a tip, you make the final amount bigger. Multiplying by 0.2 is taking twenty percent ***of*** the bill, but multiplying by 1.2 is adding twenty percent ***to*** the bill. There's a big difference between those two concepts! If you analyze the wrong answer choices, you can see that the test makers expected you to make this mistake. Options A, B, and D all use the multiplier 0.2. Only option C multiplies by 1.2. **C is the correct answer.**

Another potential point of confusion in this problem is the use of function notation in the answer choices. When some of my students look at the correct option, $t(x) = 1.2(2x + 1)$, they interpret $t(x)$ to mean *t* times *x*. This is not the case! $t(x)$ should be read as "t of x." It means that *x* is the input and *t* is the output. You can tell that *t* isn't being multiplied by *x* because of the use of the word *function* in the question. Also, it wouldn't make any sense to multiply the total bill by the price of Larry's meal.

Strategy #1 Create Models involving Totals

My last question about Harry and Larry would fall under this category. Because the concept of total was involved, I had to add up the individual amounts that Harry and Larry paid. However, that question was more complicated because it also involved percentages and function notation. Here's a more straightforward question about a total. It's pretty easy, so expect to see a question like this as one of the first five in any math section.

Example: One weekend, Ellie listened to a 6 songs per hour for *m* hours on Saturday and 4 songs per hour for *n* hours on Sunday. Which of the following represents the total number of songs Ellie listened to on Saturday and Sunday?

A) $10mn$

B) $24mn$

C) $4m + 6n$

D) $4n + 6m$

Explanation: Focus on the word ***total*** in this question. It means you have to add together the songs Ellie listened to on Saturday and Sunday. Eliminate choices A and B because they don't involve addition. In order to make sure you don't mix up your variables, read the question a second time, paying close attention to which days have which variables. Saturday has both a 6 and an *m*, and Sunday has both a 4 and an *n*. Breathe and take ten more seconds to look over the question and your work before bubbling in your answer. **D is correct**.

Try it yourself:

1.) Steve is a professional photographer. He charges $1200 to do the photography for a wedding, $500 for a birthday party, and $130 for in home family photos. In the first week of January, Steve does the photography for 1 wedding, 2 birthday parties, and visits 4 homes for family photos. If *w* is used to represent the number of weddings, *p* is used to represent the number of birthday parties, and *f* is used to represent the number of family photos, which of the following represents Steve's income in the first week of January?

A) $1200w + 500p + 130f$

B) $w + 2p + 4f$

C) $1200w + 1000p + 520f$

D) $2720wpf$

Strategy #2: Analyze models in the form y=mx+b

Hopefully you're familiar with the formula $y = mx + b$ from your Algebra One class. It is the formula for a line, where *m* is the slope, *b* is the y-intercept, and every point on the line can be written in the form (*x*, *y*). Since a line only has one slope and one intercept, *m* and *b* are referred to as constants. However, since the line goes through lots of different values of *x* and *y*, they are referred to as the variables. You can expect to see multiple questions on the SAT that ask you to analyze real world situations that can be modeled this way.

Example: Isabelle is a freelance writer. In order to meet her income goal, she needs to write a certain number of articles in a year. The number of articles that she has left to write for the remainder of the year can be represented with the equation $A = 300 - 1.2d$, where A is the number of articles remaining, and *d* is the numbers of days she has worked this year. What is the meaning of 300 in this equation?

A) Isabelle will complete all her articles in 300 days.

B) Isabelle starts each year with 300 articles to write.

C) Isabelle writes articles at a rate of 300 per month.

D) Isabelle writes articles at a rate of 300 per week.

Explanation: In this situation, the model $y = mx + b$ has been rearranged to look like $y = b - mx$. Don't let that throw you off: this is a still a linear equation. The intercept is 300 and the slope is -1.2. The slope will always be the number that is being multiplied by a variable, and the intercept will be the number that is a term by itself. You should think of an intercept as a starting point and the slope as a rate. (A rate will usually involve the word ***per***, as in miles per hour, or articles per day.) This model is calculating the number of days ***remaining***, which means the starting point is how many articles Isabelle has to write in a year, and that number is decreased by 1.2 every day she works. It looks like she starts the year with 300 articles to write, and she is writing 1.2 per day. **B is the correct answer**.

Let's use the technique of analyzing the units to double check our work. The units for the y-intercept must be the same as the units of the y-value. Since $A = 300 - 1.2d$, *A* and 300 must have the same units. The questions states that *A* is a number of articles. Now look at the answer choices again. Only B gives 300 as a number of articles.

Try it Yourself:

2.) A reading specialist uses the formula $r = 2.3a - 15.2$ to estimate the reading level of a child between 7 and 16 years of age, where *r* is the reading level and *a* is the age in years. Based on this model, what is the estimated increase in reading level each year?

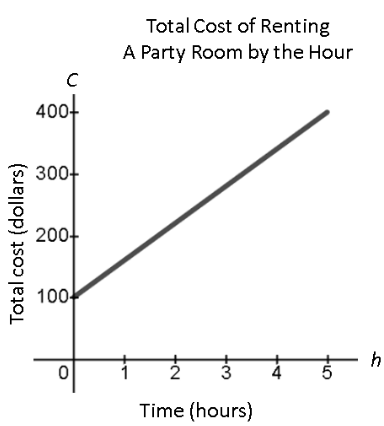
A) 2.3

B) 6.2

C) 10.8

D) 15.2

3.) Amanda is planning a party for her parents 50th wedding anniversary and decides to rent a party room at their favorite restaurant. The graph below represents the total cost *C*, in dollars, of renting the room for *h* hours.



What is the real world meaning of the *C* intercept in this graph?

A) The initial cost of renting the room

B) The total number of guests at the party

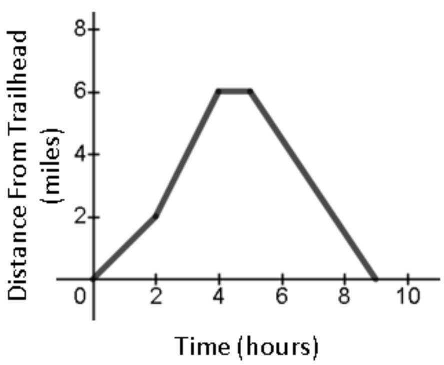
C) The total number of hours for which the room is rented

D) The increase in cost of renting the room for each additional hour

Strategy #3: Modeling real world situations with graphs

Questions of this type ask you to interpret models that are displayed as graphs rather than as equations. Each one should be displayed on the standard *xy*-plane, but the real world concepts that *x* and *y* represent can be different each time. Because of this, pay close attention to how the *x* and *y* axes are labeled. Just like the last two strategies, you shouldn't have to do any fancy calculations or series of steps. The trick is to read closely and think it through carefully.

Example: Nabob is spending the day hiking. He parks his car at a trailhead and hikes on the trail for several hours. He then stops for lunch and hikes back to his car. The graph below models his trip.



During which interval did Nabob stop for lunch?

A) First two hours

B) Between the second and forth hours

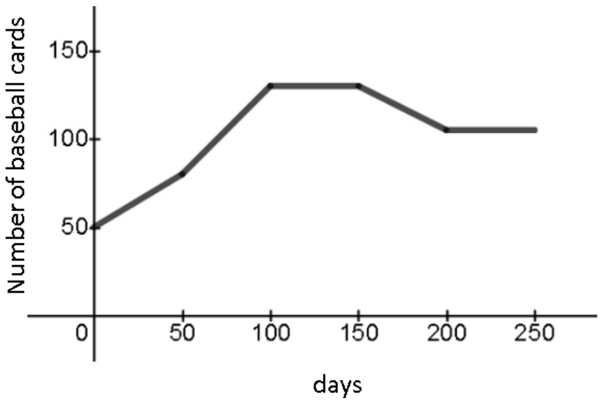
C) Between the fourth and fifth hours

D) Between the eighth and ninth hours

Explanation: Pay close attention to the label on the y-axis. This graph shows how far Nabob is from the trailhead where he parked his car throughout his trip. For the first four hours Nabob's distance from the trailhead is increasing, so he is hiking away from the trailhead. At the top of the graph, you can see a horizontal section. This is where the distance from the trailhead is not changing, so he is not moving. It is in this horizontal area that he has stopped for lunch. The rest of the graph has a negative slope, so the distance from the trailhead is decreasing. During this time, he is hiking back to the trailhead where he parked his car. For your final answer, focus on the horizontal section and notice where the it begins and ends with respect to the x-axis. To be as accurate as possible, draw vertical lines from the start of the horizontal section to the x-axis and the end of the horizontal section to the x-axis. These lines should intersect the x-axis at approximately 4 and 5. **The final answer is C.**

Try it yourself:

4.) Alex has a hobby of buying and selling baseball cards on Ebay. The following graph shows the number of baseball cards in Alex's collection over a time interval of 250 days.



During this interval, when was Alex selling more cards than he was buying?

A) Day 0-50

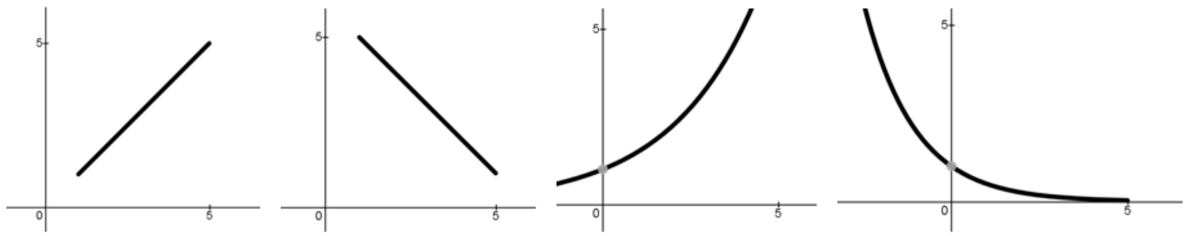
B) Day 100-150

C) Day 150-200

D) Day 200-250

Strategy #4: Recognizing the difference between linear and exponential models

I've seen several questions on the SAT that require you to differentiate between these four types of functions: linear increasing, linear decreasing, exponential growth, and exponential decay. Moving along the function from left to right, linear increasing is a line with a positive slope. Linear decreasing is a straight line with a negative slope. Exponential growth is a curve that starts almost horizontally and becomes nearly vertical. Exponential decay is a curve that starts with a near vertical drop and then flattens out.



Linear Increasing | Linear Decreasing | Exponential Growth | Exponential Decay

On the SAT you should also be prepared to identify these patterns when they are presented to you as equations or tables.

Type of Function	**Type of equation**	**Table (when the x values are increasing and equally spaced)**	**Slope (calculate the slope when the x values are not equally spaced)**
Linear Increasing	$y = mx + b$	The same number is *added* to arrive at each subsequent y value.	positive, and same value no matter which two points you use to calculate
Linear Decreasing	$y = -mx + b$	The same number is *subtracted* to arrive at each subsequent y value.	negative, and same value no matter which two points you use to calculate
Exponential Growth	$y = c^x$ $where\ c > 1$	The same number is *multiplied* to arrive at each subsequent y value.	positive, but starts as a small slope and becomes large
Exponential Decay	$y = c^x$ $where\ 0 < c < 1$	The same number is *divided* by to arrive at each subsequent y value.	negative, but starts as very steep slope and gets closer to zero

Example: Which of the following options best describes the pattern in the table below?

x	y
2	20
3	30
4	45
5	67.5

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

Explanation: After your initial scan of the question and the answer choices, your first job is to inspect the column of x values. Are they following a regular pattern? They sure are: each x value is exactly one more than the previous x value. Now that we've established that, it's useful to also look for a pattern in the y column. Are the y-values growing when the x values grow, or are they shrinking when the x values grow? Both the x and y values are getting bigger, so we can eliminate options B and D. Finally, find the difference between each y value and the one after it.

x	y	**Difference from last y value**
2	20	--
3	30	10
4	45	15
5	67.5	22.5

If the differences between the y values were all exactly the same, we would have a linear function. However, in this example we can see that the difference is getting bigger each time. This is exactly what you would expect from an exponential growth function. **The correct answer is C**.

Example: Which of the following options best describes the pattern in the table below?

x	y
-1	3
5	-3
0	2
2	0

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

Explanation: Just like the last example, we'll begin by looking for a pattern in the x values. This time, the x-values aren't in any particular order, so we can't expect to find a pattern in the y values either. Instead, we will use the slope formula to find out whether there is a consistent slope between all these points. Pick whichever two points you like and calculate the slope. Then pick a different combination of points and calculate that slope. For my example, I'll just pick the first two points and then the first and third points.

Show your work like this:

$$slope\ between\ first\ two\ points = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{5 - (-1)} = \frac{-6}{6} = -1$$

$$slope\ between\ first\ and\ third\ points = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{0 - (-1)} = \frac{-1}{1} = -1$$

It looks like the two slopes are the same, so this is a linear function. Furthermore, the slopes are equal to a negative number, so this is decreasing. **The correct answer is B**.

Try it yourself:

5.) Which of the following options best describes the pattern in the table below?

x	y
3	100
4	50
5	25
6	12.5

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

6.) Which of the following options best describes the pattern in the table below?

x	y
2	9
-1	$\frac{1}{3}$
0	1
3	27

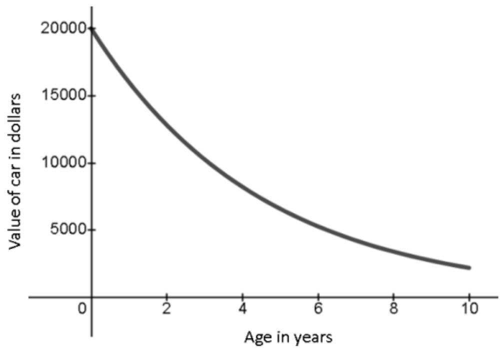
A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

7.) The graph below is a model for the value of Eric's new car for over a time period of 10 years, beginning on the day he purchased it.



Which of the following best describes this mathematical model?

A) Increasing Linear

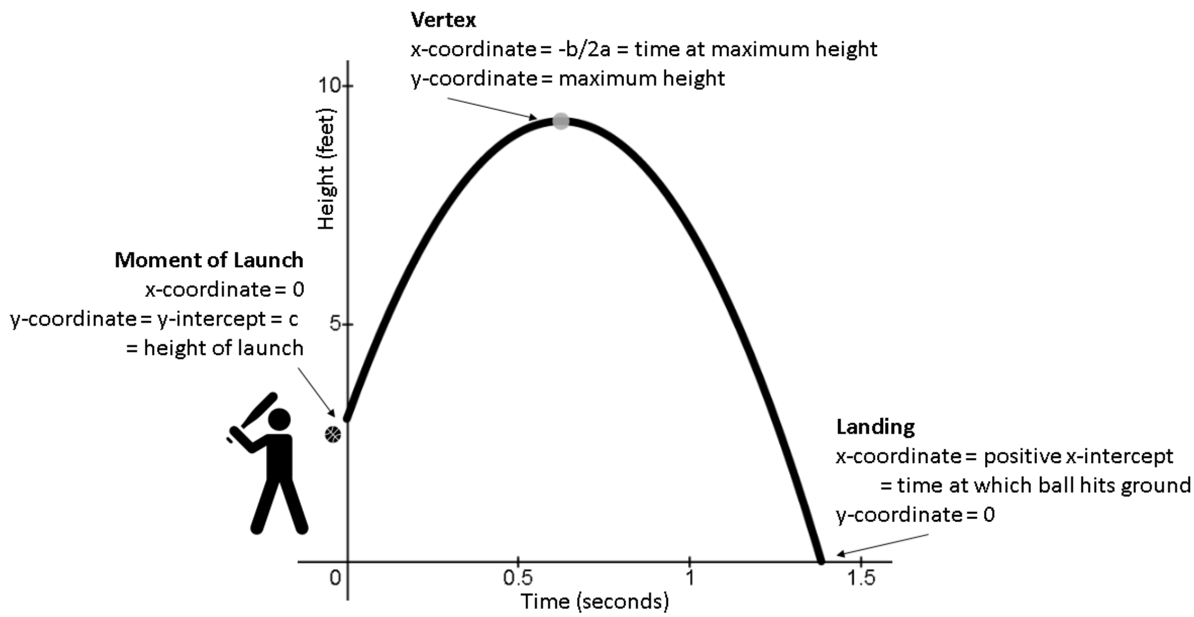
B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

Strategy #5: Analyzing parabolic motion

If you've ever kicked a soccer ball or played catch with a friend, then you're familiar with the path an object takes as it moves through the air. It follows a nice, smooth arc that physicists call parabolic motion. If you graph that nice, smooth arc on an xy-plane, it will look like a parabola with a negative leading coefficient. If you've already had physics, then you probably know exactly what I'm talking about. However, if you haven't had physics, know that SAT questions about parabolic motion are 100% solvable with what you learned in Algebra 2.



The graph above shows the trajectory of a baseball. The equation I used to make the parabola was $y = -16x^2 + 20x + 3$. This equation follows the standard model $y = ax^2 + bx + c$. If you've taken physics, your teacher should have explained that the *a* value of -16 has to do with the downward pull of gravity, the *b* value of 20 has to do with the upward velocity at which the ball was hit, and the *c* value of 3 has to do with the initial height at which the ball was hit. For the SAT, you need to know the real world meanings of both the x and y coordinates of three important points on this trajectory: the moment of launch, the vertex, and the landing.

Moment of launch: This is the moment that the football is released by the quarterback, the cannonball is shot out of the cannon, or the pumpkin is launched during the pumpkin smashing contest. If this point is plotted on the *xy*-plane, it will be the *y*-intercept. That means the *x*-coordinate will be equal to zero (zero seconds from the moment of launch) and the *y*-coordinate will be the height above the ground at which the object was released. The *y*-coordinate at this point will also be equal to the c value in the equation.

Vertex: This point is at the very top of the parabola. It is the moment at which the downward pull of gravity starts to overtake the inertia from the initial launch, and the upward motion changes to downward. The *y*-coordinate gives you the maximum height the object will reach. The *x*-coordinate gives you the time at which that maximum height is reached. If you are asked to calculate that time, use the formula $x = \frac{-b}{2a}$. Once you have the *x* value, plug it back into the equation to find the *y* value.

Landing: For mathematical models involving parabolic motion, the x-coordinate is a measurement of time, not distance. However, the x-axis does line up with the horizontal line $y = 0$, so the *x*-axis can be thought of as the ground. In other words, the positive x-intercept is the moment the projectile first hits the ground. If there's any bouncing or rolling, that would happen after the initial landing, and would be beyond the scope of any SAT question. Also, don't be confused by the word "positive" as in positive x-intercept. It just means that if you were to draw the whole parabola, you would see it also intersects the x-axis behind the point of launch. Mathematically, you can draw and calculate this negative *x*-intercept, but in the real world it's something that never actually happens. If you need to calculate the exact time at which the projectile hits the ground, you will need to set y equal to zero, and then either factor or use the quadratic formula to solve for x.

Example: The equation $h = -16t^2 + 81t$ represents the height of a water-powered rocket, in feet, *t* seconds after it is launched vertically into the air at a velocity of 81 feet per second. After approximately how many seconds will the rocket hit the ground? (You can ignore the effect of air resistance for this question.)

A) 3.5

B) 4.0

C) 4.5

D) 5.0

Explanation: This question is asking you to calculate something that has to do with the landing point of the rocket's trajectory. You know for sure that when the rocket hits the ground, the *y*-coordinate will be equal to zero. In other words, you're looking for the positive *x*-intercept. Don't be thrown off by the fact that this question has used *h* where there is usually an *h* and *t* where there is usually an *x*. That doesn't change your work at all. Now that you've interpreted the real world meaning of the question, we can solve the problem. Substitute 0 for *h* and solve for *t.*

Show your work like this: $0 = -16t^2 + 80$

$0 = t(-16t + 81)$

$0 = t$ and $0 = -16t + 81$

$0 = t$ and $t = \frac{81}{16} \approx 5.06$

Ignore the first answer of $t = 0$. That is the launch point, not the landing point. The rocket lands after approximately 5 seconds. **D is the correct answer**.

Try it yourself:

8.) The quadratic function $f(x) = -4.9x^2 + 20x + 1.3$ models the height above the ground *f(x)*, in meters, of a projectile *x* seconds after it was launched into the air. If this model was graphed on the *xy*-plane, which of the following would represent the real world meaning of the positive *x* intercept of the graph?

A) The time at which the projectile lands on the ground

B) The height from which the projectile was initially launched

C) The maximum height of the projectile's trajectory

D) The time at which the projectile reaches its maximum height

9.) The quadratic function $f(x) = -4.9x^2 + 9x + 2.4$ models the height above the ground *f(x)*, in meters, of a projectile *x* seconds after it was launched into the air. If this model was graphed on the *xy*-plane, which of the following would represent the real world meaning of the value 2.4?

A) The time at which the projectile lands on the ground

B) The height from which the projectile was initially launched

C) The maximum height of the projectile's trajectory

D) The time at which the projectile reaches its maximum height

10.) The quadratic function $f(x) = -16x^2 + 5x$ models the height above the ground *f(x)*, in meters, of a projectile *x* seconds after it was launched into the air. Which of the following is the closest to the maximum height, in meters, of the projectile's path? *(Hint: use the formula x=-b/2a , and be careful to avoid the trap.)*

A) 0.15

B) 0.31

C) 0.39

D) 3.2

Strategy #6: Analyzing effect of increasing or decreasing one variable

The final strategy I want to discuss in this chapter has to do with analyzing a mathematical model to determine what will happen to one variable when another one is changed. There are usually two ways to approach these problems: pure analysis and number picking. Pure analysis may come across as simply "looking" at the question and then magically knowing the answer, but there are mental steps that you follow to accomplish that. Number picking will definitely take longer, but you may feel more confident about your final answer. I'll walk you through both methods, and you can pick the one that works better for you.

Example:

$$\mathrm{I} = \frac{P}{4\pi r^2}$$

The intensity of a sound heard by an observer can be calculated by the model above, where I is the intensity (or loudness) of the sound to the observer, P is the power of the sound at its source, and r is the distance between the source and the observer. If Bryan is standing three times as far from the source of a sound as Renee, how many times louder will the noise sound to Renee as it does to Bryan?

A) 3

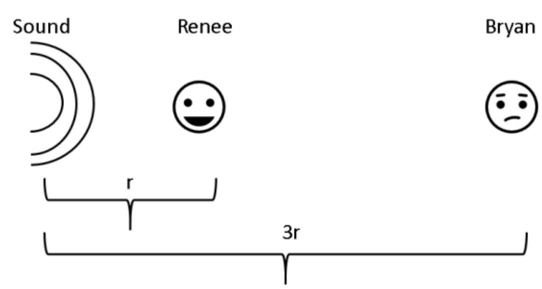
B) 4π

C) 9

D) 12π

Explanation #1: Pure analysis

There's a lot going on in this question, so I'm going to make a quick drawing to help me understand the situation.



Logic tells me that that since Bryan is standing further away, the sound intensity should be lower for him and higher for Renee. Unfortunately, all the options have the sound as louder for Renee, so we can't eliminate anything at this point. Next, looking carefully at the formula, I can see that *r* needs to be replaced with 3*r*.

$$? = \frac{P}{4\pi(3r)^2}$$

I recommend you write this formula out and be especially careful to put *3r* in parentheses. It looks like that three will be squared, so there will be a 9 affecting the other side of the equation. Also, we're making the denominator bigger, so it makes sense that the intensity for Bryan will get smaller. A is a tricky trap: it's for people who didn't notice that the factor of 3 would need to be squared. B and D are also tempting because there's a π in them, and that just makes them look like smart person choices. However, when we triple the *r* value, we're not putting in an extra π, so there won't be a new factor of π on the other side either. **C is the correct answer.**

Explanation #2: Number Picking

This method will take a little more time and number crunching, but it will feel more concrete and dependable. Let's pick numbers for all the variables except for *I*, and then use those numbers to calculate *I*. We will have to go through the process twice: once for Renee and once for Bryan

Renee: Pick $P = 10$ and $r = 2$. Using the formula, we get $I = \frac{10}{4\pi(2)^2} \approx 0.199$

Bryan: Pick $P = 10$ and $r = 6$. Using the formula, we get $I = \frac{10}{4\pi(6)^2} \approx 0.022$

Compare the results by dividing them: $\frac{0.199}{0.022} \approx 9.04$

Ignoring our rounding error, the sound is 9 times louder for Renee. **C is the correct answer**. Don't believe me? Try picking different numbers. As long as you use the same value of *P* for both people and exactly three times the *r* value for Bryan, you will get a result very close to 9.

Try it yourself:

11.) At the Hamilton Family Diner, *n* cups of coffee are made by adding *s* scoops of ground coffee to the coffeemaker. If $n = 2s + 3$, how many additional scoops are needed to make each additional cup of coffee?

A) 0.5

B) 1

C) 2

D) 3

12.) The speed of sound traveling through salt water can be given as a linear function with respect to the temperature of the water. The function is $s(t) = 4410 + 5.5t$ where s is the speed in feet per second and t is the temperature in degrees Fahrenheit. Which of the following statements is the best interpretation of the number 5.5 in this context?

A) The speed of sound through salt water at 0°F

B) The speed of sound through salt water at 5.5°F

C) The increase in speed when the temperature is raised 1° F

D) The increase in speed when the temperature is raised 5.5° F

$$F = G\frac{m_1 m_2}{r^2}$$

13.) The gravitational force that two objects exert on each other can be calculated by the formula above, where *G* is the universal gravitational constant, m_1 and m_2 are the masses of the two objects, and *r* is the distance between them. An astronomer uses the formula to find the force of gravity between two celestial objects when they are different distances apart. What is the ratio of the gravitational force when the objects are 1.5×10^6 km apart to the gravitational force when they are 3×10^6 km apart?

A)1:4

B) 1:2

C) 3:2

D) 4:1

Practice

No Calculator Questions:

Easy:

1.) The function below is best described as which of the following?

x	f(x)
6	150
8	151
10	152
12	153
14	154

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

2.) The function below is best described as which of the following?

x	f(x)
-2	540
-1	270
0	135
1	67.5

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

3.) The function below is best described as which of the following?

x	f(x)
200	3
300	6
400	12
500	24
600	48

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

4.) The function below is best described as which of the following?

x	f(x)
-20	40
-18	37
-16	34
-14	31
-12	28

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

5.) The density of an object can be found using the model $\rho = \frac{m}{v}$ where ρ is the density, *m* is the mass, and v is the volume. If an object has a density of 15 grams per cubic centimeter and a mass of 5 grams, what is its volume in cubic centimeters?

A) 1/3

B) 3

C) 5

D) 15

Medium:

6.) A model for the speed of sound in air is $v = 331 + 0.6T$ where v is the speed of sound in meters per second and *T* is the air temperature in °C. Given this model, what does the number 0.6 represent?

A) The speed of sound at an air temperature of 0° C.

B) The speed of sound at an air temperature of 331° C.

C) The increase in the speed of sound for each increase of 1° C in air temperature.

D) The increase in the speed of sound for each increase of 0.6° C in air temperature.

7.) Tarik has a newspaper route and can calculate the number of newspapers remaining to be delivered using the model $R = 336 - 48d$ where *R* is the number of newspapers he still needs to deliver in a given week and *d* is the number of days in the week he has already delivered papers. What is the best interpretation of the number 336 in this model?

A) The number of newspapers he has to deliver at the beginning of each week.

B) The number of newspapers he delivers each day.

C) The number of newspapers he delivers in 48 days.

D) The number of days he has been delivering newspapers.

8.) Eric is planting a new lawn in his backyard. He will need to purchase grass seed and rent a spreader and small bulldozer for the duration of the project. He calculates the total cost of the project, *C*, with the model $C = G + (S + B)d$ where G is the cost of the grass seed, S is the daily cost to rent a spreader, and B is the daily cost to rent a small bulldozer. What is the best interpretation for the variable, *d*?

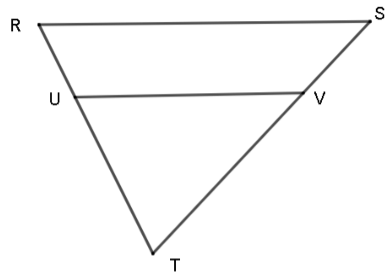
A) The model of bulldozer that Eric rents

B) The quality of grass seed that Eric purchases

C) The number of days the project takes

D) The size of Eric's lawn

Hard:



9.) In the figure above, $\overline{RS} \| \overline{UV}$. If $RU = 4$ and $UT = 7$, what is the ratio of the area of ΔRST to the area of ΔUVT?

A) $\frac{4}{7}$

B) $\frac{7}{4}$

C) $\frac{49}{16}$

D) $\frac{121}{49}$

$$F = G\frac{m_1 m_2}{r^2}$$

10.) The gravitational force that two objects exert on each other can be calculated by the formula above, where *G* is the universal gravitational constant, m_1 and m_2 are the masses of the two objects, and *r* is the distance between them. An astronomer uses the formula to find the force of gravity between two celestial objects when they are different distances apart. What is the ratio of the gravitational force when the objects are $1\ AU$ (Astronomical Units) apart to the gravitational force when they are $4\ AU$ apart?

A) $\frac{1}{16}$

B) $\frac{1}{4}$

C) $\frac{4}{1}$

D) $\frac{16}{1}$

$$I = \frac{P}{4\pi r^2}$$

11.) The intensity of a sound heard by an observer can be calculated by the model above, where *I* is the intensity (or loudness) of the sound to the observer, *P* is the power of the sound at its source, and *r* is the distance between the source and the observer. If Albert is standing five times as far from the source of a sound as Esther, how many times louder will the noise sound to Esther as it does to Albert?

A) 5

B) 4π

C) 20π

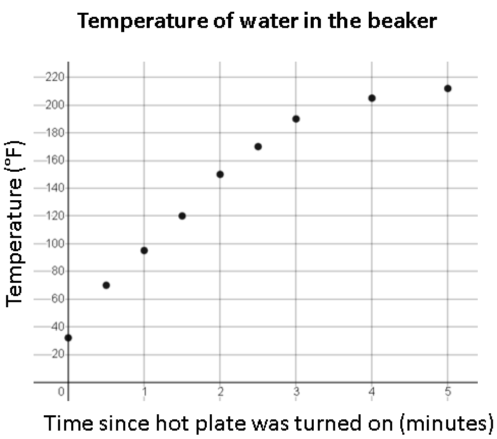
D) 25

Calculator Questions:

Easy:

Use the following information for questions 1-3.

Kaitlyn is using a hot plate to heat water in a beaker for a chemistry experiment. She measures the temperature of the water every 30 seconds while it is heating and records her data on the following graph:



1.) Of the following, which is the closest to the temperature, in degrees Fahrenheit, of the water when Kaitlyn first turns on the hot plate?

A) 33

B) 70

C) 150

D) 212

2.) During which of the following intervals does the temperature of the water increase at the greatest average rate?

A) Between 1 and 2 minutes

B) Between 2 and 3 minutes

C) Between 3 and 4 minutes

D) Between 4 and 5 minutes

3.) Mamma Rosa's famous pizza has the following nutritional facts:

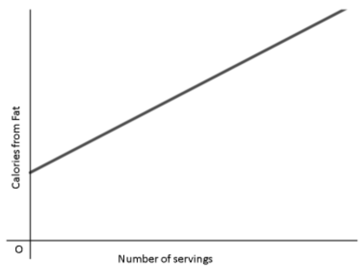
Serving Size: 6oz slice

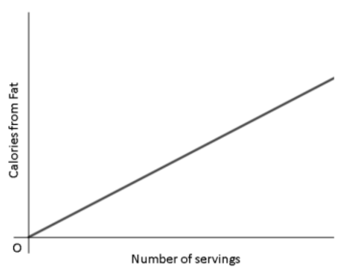
Servings per pizza: 10

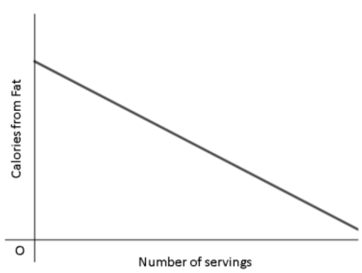
Calories per serving: 230

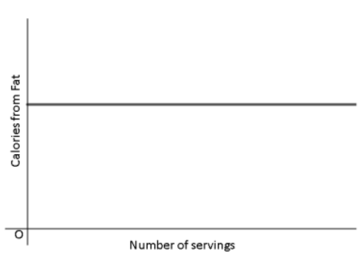
Percent of calories from fat: 60%

Which of the following could be the graph of the number of calories from fat in Mamma Rosa's pizza as a function of the number of 6 oz servings?

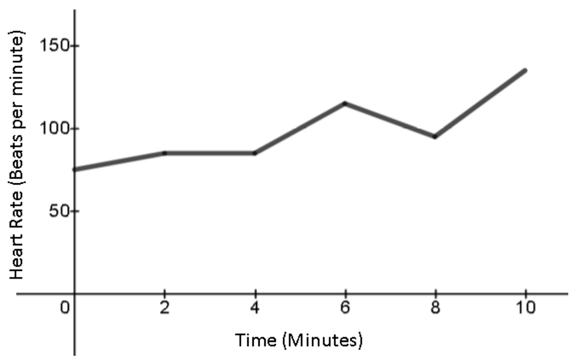
A) 

B) 

C) 

D) 

4.) An athlete's personal trainer measures her heartrate during the first 10 minutes of an exercise routine and records the data in the graph below.



During which interval is the athlete's heart rate strictly decreasing then strictly increasing?

A) Minutes 0-4

B) Minutes 2-6

C) Minutes 4-8

D) Minutes 6-10

Medium:

5.) The trunk diameter, D, in inches of an oak tree that is t years old, can be modeled by the equation

$D = \frac{2t-1}{3}$. According to the model, for every increase in 1 year of age, by approximately how many inches will the diameter of an oak tree increase?

A) 0.33

B) 0.67

C) 2.0

D) 1.5

6.) Priya is an editor for a weekly magazine. The amount of time it takes her to proofread an article can be modeled by the equation $T = 12p + 16.3$ where T is the time in minutes and p is the number of pages in the article. How much longer will it take Priya to proofread a 12 page article than to proofread a 10 page article?

A) 12 minutes

B) 16.3 minutes

C) 24 minutes

D) 28.3 minutes

7.) The line graphed in the xy-plane below models the cost of a ball bearing with respect to the quantity of ball bearings ordered.

According to the graph, by how many dollars is the cost of a ball bearing reduced for each additional ball bearing ordered?

A) 0.025

B) 0.25

C) 4

D) 6.25

8.) A contracting company purchased construction equipment valued at $630,000. The value of the equipment depreciates by the same amount each year, so that after 10 years the value will be $130,000. Which of the following equations gives the value, *y*, of the equipment *x* years after it was purchased for the time period $0 \leq x \leq 10$?

A) $y = 130000 + 50000x$

B) $y = 130000 - 50000x$

C) $y = 630000 + 50000x$

D) $y = 630000 - 50000x$

9.) The quadratic function $a(t) = -16t^2 + 22t + 4$ models the height above the ground, in feet, of a pebble *t* seconds after it was launched from a child's slingshot. If $y = a(t)$ is graphed on the xy-plane, which is the real world meaning of the *y* coordinate of the vertex?

A) The initial height of the slingshot

B) The maximum height of the pebble

C) The time at which the maximum height is reached

D) The time at which the pebble lands on the ground

10.) The function below is best described as which of the following?

x	f(x)
-2	-25
0	3
5	73
-1	-11

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

Hard:

11.) The graph of the exponential function g in the *xy*-plane, where $y = g(x)$, has a *y*-intercept of *c*, where *c* is a positive constant. Which of the following could define the function *g*?

A) $g(x) = -5(c)^x$

B) $g(x) = -c(x)^5$

C) $g(x) = c(5)^x$

D) $g(x) = 5cx$

Grid In

12.) The trajectory of a soccer ball after it is kicked from ground level can be modeled by the function $s(t) = -4.9t^2 + 16t$, where *s* is the height of the soccer ball above the ground in meters, and *t* is number of seconds after it was kicked. After it is kicked, approximately how many seconds will it take before the ball lands on the ground? (Round your answer to the nearest tenth when gridding it in.)

/	○	○	○	○
.	○	○	○	○
0	○	○	○	○
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○

Use the following information for questions 13 and 14:

A robot used for a certain manufacturing application was originally purchased for $350,000 and is depreciating at a rate of 15% per year. The manufacturing engineer uses the equation $V = 350000(r)^t$ to model the value of the robot after *t* years.

13.) What value should the engineer use for *r*?

/	○	○	○	○
.	○	○	○	○
0	○	○	○	○
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○

14.) In thousands of dollars, what does the manufacturing engineer believe the value of the robot will be after 6 years? (Round to the nearest thousand and disregard the $ sign when gridding your answer. A calculator is permitted on this question.)

/	○	○	○	○
.	○	○	○	○
0	○	○	○	○
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○

Post-Test

Directions: These six problems are similar to the six problems you tried in the pre-test. Give them a try to see how much you have improved. Use a calculator only if you need to.

 # Correct:

1.) An automobile with a price of $15,900 is to be purchased with an initial payment of $2500 and weekly payments of $268 each. Which of the following equations can be used to find the number of weekly payments, w, required to complete the purchase? (Assume there will be no taxes, interest, or fees.)

A) $15900 = 2500w - 268$

B) $15900 = 2500 - 268w$

C) $15900 = 2500 + 268w$

D) $15900 = 268w$

2.) A submarine is holding steady at a depth of 30 feet below the surface of the ocean. The submarine then begins to dive and increases its depth at a rate of 17 feet per second. Which of the following functions represents the submarine's depth *d*, in feet, *s* seconds after it begins the dive?

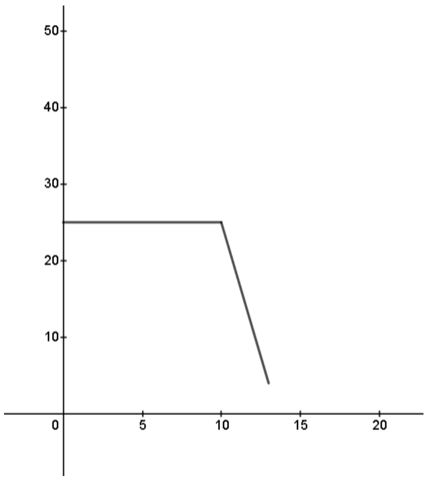
A) $d = 17s - 30$

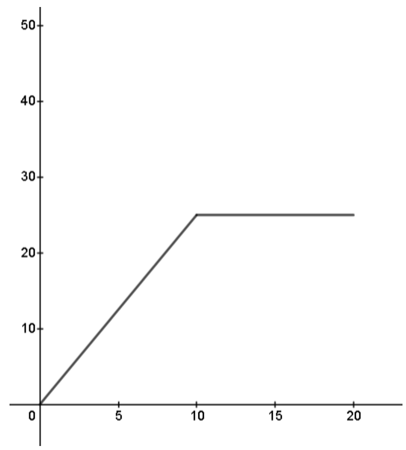
B) $d = 30 - 17s$

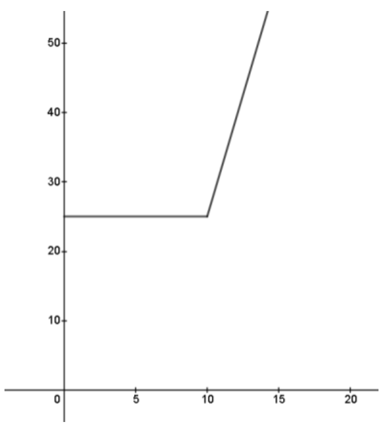
C) $d = 17s + 30$

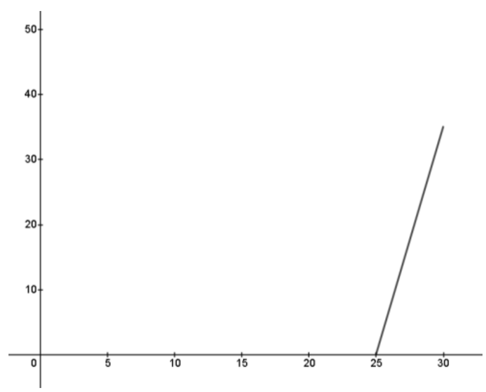
D) $d = 30s + 17$

3.) Angela's gym charges her \$25 each month, which includes the cost of 10 visits. If Angela wants to visit the gym more than 10 times in a month, she has to pay \$7 for each additional visit. Which of the following represents Angela's monthly cost of going to the gym, y, for a total of x visits, as graphed on the standard xy-plane?

A)


B)


C)


D)


4.) Which of the following options best describes the pattern in the table below?

x	y
0	3000
1	1000
2	$333\frac{1}{3}$
3	$111\frac{1}{9}$

A) Increasing Linear

B) Decreasing Linear

C) Exponential Growth

D) Exponential Decay

5.) The quadratic function $h(x) = -16x^2 + 19x - 2$ models the height of a golf ball above the ground *x* seconds after a golfer hits the golf ball out of a sand trap. If $y = h(x)$ is graphed on the standard *xy*-plane, which of the following is the real world meaning of the greater of the two x intercepts of the graph? (For this question, consider the sand trap to be a hole with a depth of 2 feet below the ground level.)

A) The time at which the golf ball reaches its maximum height

B) The maximum height of the golf ball

C) The depth of the sand trap

D) The time at which the golf ball reaches the ground

6.) Tabitha is a copyeditor for a publishing company. The amount of time it takes her to edit can be modeled by the equation $T = 8p + 16.3$ where *T* is the time in minutes and *p* is the number of pages she proofreads How much longer will it take Tabitha to edit 20 pages than it takes her to edit 15 pages?

A) 40 minutes

B) 56.3 minutes

C) 120 minutes

D) 160 minutes

Answer Key

Pre-test

1.) D

2.) A

3.) A

4.) B

5.) B

6.) A

Try it yourself

1.) A

2.) A

3.) A

4.) C

5.) D

6.) C

7.) D

8.) A

9.) B

10.) C

11.) A

12.) C

13.) D

Practice - No Calculator

1.) A

2.) D

3.) C

4.) B

5.) A

6.) C

7.) A

8.) C

9) D

10.) D

11.) D

Practice – With Calculator

1.) A

2.) A

3.) B

4.) D

5.) B

6.) C

7.) A

8.) D

9.) B

10.) A

11.) C

12.) 3.3

13.) 0.85

14.) 132

Post-test

1.) C

2.) C

3.) C

4.) D

5.) D

6.) A